Problem Set 36: Compound Interest

Calculate the final amount (accumulated value) in each case:

a) $7,000 invested for 8 years at 9% per annum compounded annually.

b) $6,350 invested for 11 ½ years at 8% per annum compounded semi-annually.

c) $9,000 invested for 7 years at 10% per annum compounded quarterly.

d) $15,000 invested for 15 years at 12% per annum compounded monthly.

e) $10,000 invested for 8 years at 9% per annum compounded weekly.

f) $14,000 invested for 50 years at 4 ¾% per annum compounded daily.

g) $14 million invested for 1 month at 4% per annum compounded monthly.

h) $1 million for 1 month @ 4%/a c. m.

Answers:
a) $13,947.94 b) $15,650.94 c) $17, 968.46 d) $89,937.03 e) $20,531.55 f) $150, 490.93
g) $14,046,666.67 h) $1,003,333.33

Problem Set 37: Present Value

1. Mrs. Fangrad borrowed $9,500 for 3 years at 11.6% per annum, compounded quarterly.
   a) How much money did she owe at the end of 3 years?
   b) How much interest did she pay for the loan?
2. What is the Present Value of each amount?
   a) $9,000 in 5 years, invested at 11% per annum, compounded semi-annually.
   b) $50,000 in 9 months, invested at 11% per annum, compounded quarterly.
   c) $100,000 in 3 years, invested at 3% per annum, compounded monthly.
   d) $78,840 in 9 years, invested at 4.8% per annum, compounded annually.
   e) $250,000 in a year, invested at 8.75% per annum, compounded quarterly.
3. Sue wants to provide for her niece's education. How much should she invest on the day her niece is born to have $22.000 on her 18th birthday, if the money earns 7% per annum, compounded quarterly?
4. Samantha wants to have $40,000 available for a down payment on a house in 10 years. How much should she invest now at $6\frac{1}{4}$% per annum, compounded semi-annually?

Answers:
1 a) $13,387.77 b) $3,887.77 2 a) $5,268.88 b) $46,091.89 c) $91,403.38 d) $51,700.54
e) $229,270.89 3. $22,000 4. $21,616.27

Problem Set 38: Rates

1. $4,000 doubles in 5 years. Calculate the annual rate of interest, compounded annually.
2. $5,500 is invested for $7\frac{1}{2}$ years and accumulates to $11,434,10. Calculate the annual rate of interest compounded semi-annually.
3. $7,000 is tripled in 11 years. Calculate the annual rate of interest, compounded quarterly.
4. $3,850 is invested for 12 years and accumulates to $14,325.77. Calculate the annual rate of interest compounded monthly.
5. $1 million is invested for 20a and accumulates to $5 million. Calculate the annual rate of interest compounded weekly.
6. $5 is invested for 100 years and accumulates to $20,000. Calculate the annual rate of interest compounded daily.
7. $5,000 is invested for 9 years at $8\frac{1}{2}$% per annum compounded daily. If an investment of $4,000 yields the same accumulated value in 8 years, calculate the annual interest rate compounded annually.

Answers:
1. 14.9% 2. 10.0% 3. 10.1% 4. 11.0% 5. 8.1% 6. 8.3% 7. 13.1%

Problem Set 39: Rates

1. How long will it take for $3,000 to accumulate to $7,000 at each rate?
   a) 7% per annum compounded annually
   b) 9% per annum compounded semi-annually
   c) 8% per annum compounded quarterly
   d) 9% per annum compounded monthly
   e) $7\frac{3}{4}$% per annum compounded minutely
2. How long will it take for $2,700 to triple at 8% per annum compounded daily?
3. At what annual rate compounded semi-annually will $2,700 triple twice as fast as in #2?
4. At a certain rate of simple interest and a given amount of time, $500 will accumulate to $750. Calculate the accumulated value if $500 is invested at twice the rate of simple interest and three times as long.

Answers:
1 a) 12.52a or 12a27w b) 9.62a or 9a226d c) 10.697a or 10a255d d) 9.450a or 9a1644 e) 10.933a or 10a11m
2. 13.734a or 13a38w 3. 16.7% 4. $2,000

Problem Set 40: Ordinary Annuities

1. Find the amount of each investment.
   a) $1,500 at the end of each year, for 6 years, at 7.1% per annum, compounded annually.
   b) $300 at the end of each 6 months, for 12 years, at 4.95% per annum, compounded semi-annually.
   c) 36 monthly payments of $100 at the end of each month, for 3 years, at 6% per annum, compounded monthly.
2. Find the payment for each ordinary annuity.
   a) 20 semi-annual payments totalling $10,000 at 6% per annum, compounded semi-annually.
   b) An amount of $7,000 with payments every 3 months for 5 years at 6.15% per annum, compounded quarterly.
   c) 36 monthly payments totalling $4,000 at 7% per annum, compounded monthly.
3. For each investment, how much must be deposited now to receive 12 payments of $1,000
   a) 6% per annum, compounded annually, with annual payments, starting in one year
   b) 6% per annum, compounded semi-annually, with a payment every 6 months, starting in 6 months
   c) 6% per annum, compounded quarterly, with a payment every 3 months, staring in 3 months
   d) 6% per annum, compounded monthly, with monthly payments, staring in a month

Answers:
1. a) $10,757.01 b) $9,674.76 c) $3,933.61 2. a) $372.16 b) $301.60
c) $100.18 3. a) $8,383.83 b) $9,954.00 c) $10,907.51 d) $11,618.93

Problem Set 41: Applications

1. David is planning to start saving for his pension by making the same deposit every 6 months starting 6 months after his 35th birthday. The plan he has chosen earns 9% per annum, compounded semi-annually. How much does each regular deposit need to be in order to have $\frac{1}{2}$ million on his 60th birthday?
2. Shannon plans to buy a new tractor in 3 years. Based on current prices, she predicts a new tractor , including taxes will cost $90,000 in 3 years. How much should she invest at the end of each month at 9% per annum, compounded monthly, to have enough money to buy a tractor in 3 years?
3. At the end of grade 9, Rahid set up a annuity to save for university. At the end of each month, he invests $200 into an account bearing interest at 6.25% per annum, compounded monthly. How much money will he have at the end of grade 12?

(Problem Set 41 continued)

4. Nelida is purchasing a car for $30,000, including taxes. She hopes to replace it in 4 years with a similar car. She estimates that in 4 years, the price will have increased 25%, and her present car will have lost 60% of its value. GST of 7% is charged on the difference between the trade-in value and the new car price. PST of 8% is charged on the price of the new car. She will start saving in 3 months, by making a payment every 3 months into an account paying 8% interest per annum, compounded quarterly.
   a) How much should each payment be so that she can pay cash for the new car in 4 years?
   b) Explain assumptions you made when finding the payment, and give your opinion about the importance of the assumptions.
5. A lottery to raise funds for a hospital is advertising a $240,000 prize. The winner will receive $1,000 every month for 20 years, staring a year from now.
   a) If the interest rate is 8.9% per annum, compounded annually, how much must be invested to have the money to pay this prize?
   b) If they were able to negotiate an interest rate of 9.3% per annum compounded annually, how much would they need to invest?
6. When Jodi's grandmother retired, she decided to invest some money so she would receive $10,000 every six months for 10 years, starting in half a year. Her investment plan pays interest at 5.9% per annum, compounded semi-annually.
   a) How much must she invest?
   b) Draw a time line to illustrate the investment. Explain how the time line supports your answer.
7. Cora received an insurance settlement of $80,000, which she invested at 5.2% per annum, compounded monthly, to provide payment each month for ten years, starting next month.
   a) How much will each regular payment be?
   b) How much did Cora's insurance settlement give he altogether?
8. Wray bought a bicycle for $2,500, plus GST and PST, to compete in triathlons. He arranged to make a payment to the store at the end of every month for 2 years. The store is charging 11% interest per annum, compounded monthly.
   a) How much is each payment?
   b) How much interest is Wray paying?
9. Marvins graduating class raised $2,198.74 to establish a fund for a scholarship of $200, staring the next year, for the student who contributed most to the school. The money is invested at 4.8% per annum, compounded annually. For how many years can this scholarship be awarded before the money runs out?

Answers:
1. $2,801.07 2. $2,186.98 3. $7896.71 4. a) $1,624.79 b) rate stays at 8% for 4 years 5. a) $856.37 b) $108,800.92
6. $149,464.83 7. a) $856.37 b) $102,764.40 8. a) $134 b) 341 9. for 16 years

Problem Set 42: Time

1. Both Bob and Jack wish to save up $2 million each for retirement. Bob decides to start investing as soon as he starts working and negotiates 7.75% per annum compounded monthly for 40 years. Jack, not taking investing as seriously, decides to start once he is married. He negotiates 7% per annum compounded monthly for 27 years.
   a) Compare their monthly payment.
   b) Calculate the interest earned on each investment.
2. How long will it take in each case?
   a) $500 to grow to $5000 if invested at 10.5% per annum compounded monthly
   b) $500 to grow to $5,000,000 at 18.7% per annum compounded daily
   c) $1.02 to grow to $15 at 2.5% per annum compounded weekly
   d) $1273.25 to grow to $20,000 at 10.75% per annum compounded quarterly
3. How long will it take?
   a) $500 per month at 4.9% per annum compounded monthly to grow to $2 million
   b) Mr. F has a nest egg of $1.5 million at the time of his retirement. He expects to have $5,000 per month to live on. How long will his nest egg last if invested at 5.3% per annum compounded monthly?
   c) If you currently have $500,000 and you wnat $2500 per week to live on, how long will the money last if invested at 12% per annum compounded weekly?
   d) How long will it take for $2 per day invested at 1.5% per annum compounded daily to grow to $2,000?

Answers:
1. $615.73; $1,704.50 2. a) 22.025a or 22a9d b) 49.27a or 49a3m c) 107a28w d) 25a50w
3. a) 58.3357a or 58a123d b) 21.253a or 21a92d c) 5.165a or 5a60d d) 2.684a or 2a250d

Problem Set 43: Car Loans

1. Sue and her friend Lola decide to purchase the same kind of vehicle for $78,572.78. Both receive the same loan rate of 5.5% per annum compounded monthly. Sue agrees to pay back the monies owed in 5 years while Lola agrees to pack pack the monies owned in 3 years.
   a) Calculate the monthly payment for Sue and the monthly payment for Lola.
   b) Draw up a spreadsheet (as demonstrated in class) for the first 5 payments made by Lola.
   c) Compare the total interest paid (using formulas) by Sue and by Lola.

Answers:
1. a) S - $1500.83; L - $2372.58 b) see overhead c) S - $11,477.02; L – $6,840.10

Problem Set 44: Mortgages

1. Create a spreadsheet for the question demonstrated in class, but change the interest rate to a high interest rate of 15% per annum.
2. Consider a $500,000 dream home amortized over 20a @ 5.25% compounded semi-annually
   (a) Calculate the monthly payment amount
   (b) Draw up a repayment schedule for the first year of payments
   (c) Calculate for the 20 years:
      i. OOP
      ii. Interest

Answers:
1. see overhead 2. a) $3,353.45 b) see overhead c) $804,828; $304,828

Problem Set 45: Mortgages - Time and Money Saved

1. Consider a $300,000 home with a 5% down payment borrowed at 6.5% per annum compounded semi-annually over 25 years. Compare the total OOP and total interest **for each** of the following payments:
   (a) monthly
   (b) bi-weekly
   (c) weekly
   What conclusion can you come to about the frequency of payments?

Answers:
Monthly: payment - $1909.00; OOP - $572,699.34; Int. - $287,699.34
Bi-weekly: payment - $879.81; OOP - $571,876.50; Int. - $286,876.50
Weekly: payment - $439.64; OOP - $571,532; Int. - $286,532